جامعة بنها

كلية الحاسبات والذكاء الإصطناعي

Discrete Mathematics

Lecture 06

Dr. Ahmed Hagag

Faculty of Computers and Artificial Intelligence
Benha University

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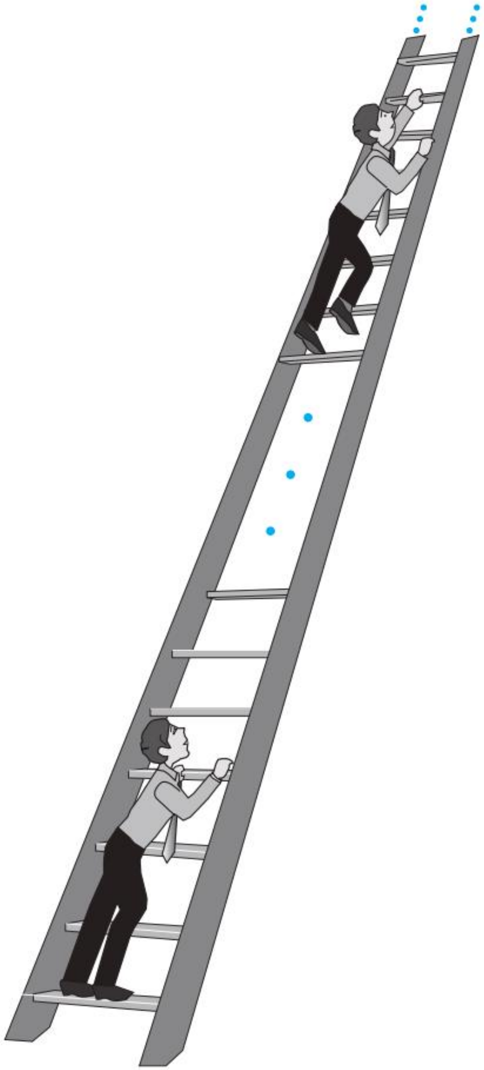
Thanks!

ياسمين وائل لبيب شادى
عبدالله محمد خالد عبدالباسط هيكل

Chapter 5: Induction and Recursion

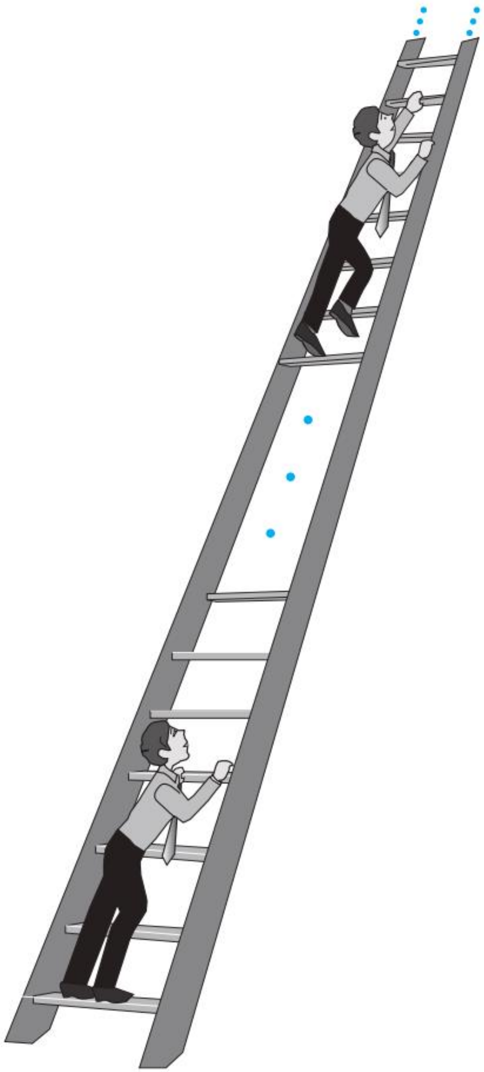
- Mathematical Induction.
- Recursive Definitions.

Mathematical Induction (1/10)



Infinite ladder

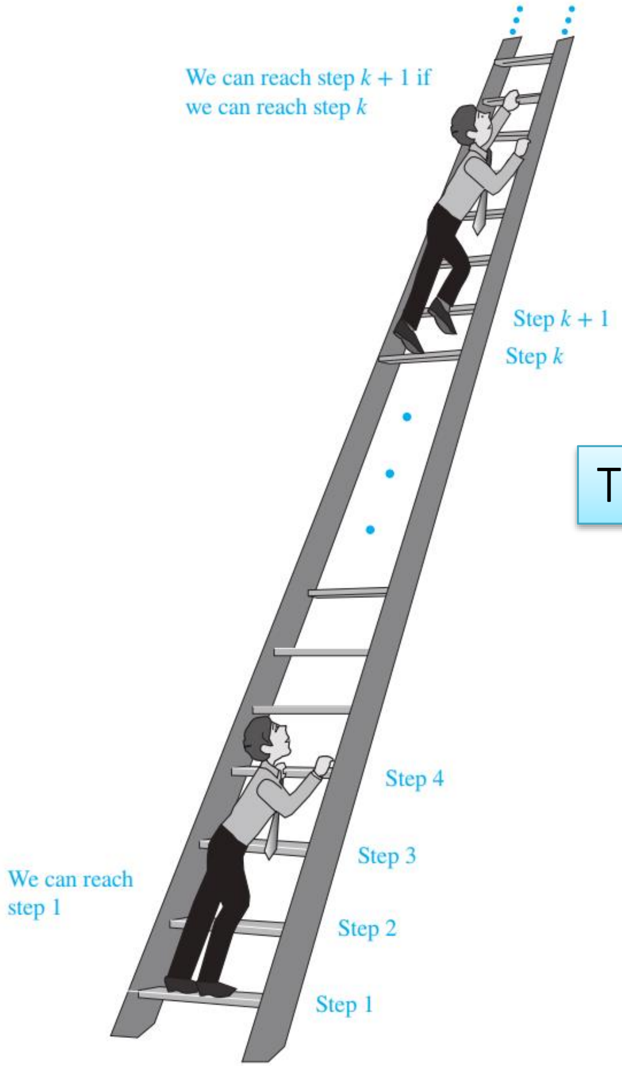
Mathematical Induction (1/10)



Infinite ladder

1. We can reach the first rung of the ladder.
2. If we can reach a particular rung of the ladder, then we can reach the next rung.

Mathematical Induction (1/10)

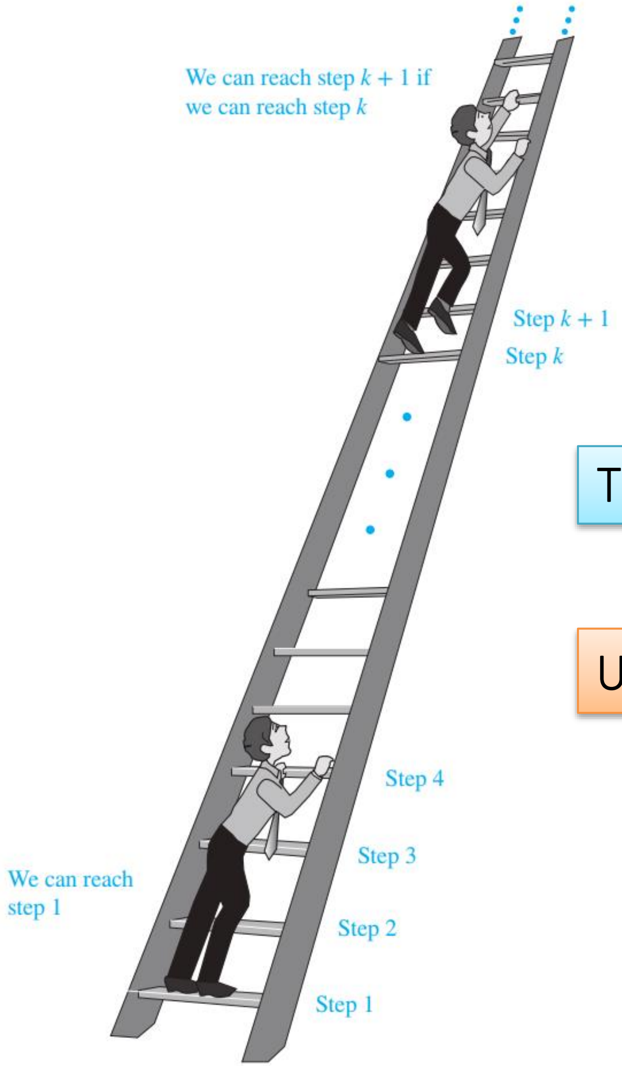


Infinite ladder

1. We can reach the first rung of the ladder.
2. If we can reach a particular rung of the ladder, then we can reach the next rung.

Therefore, we are able to reach every rung of this infinite ladder

Mathematical Induction (1/10)



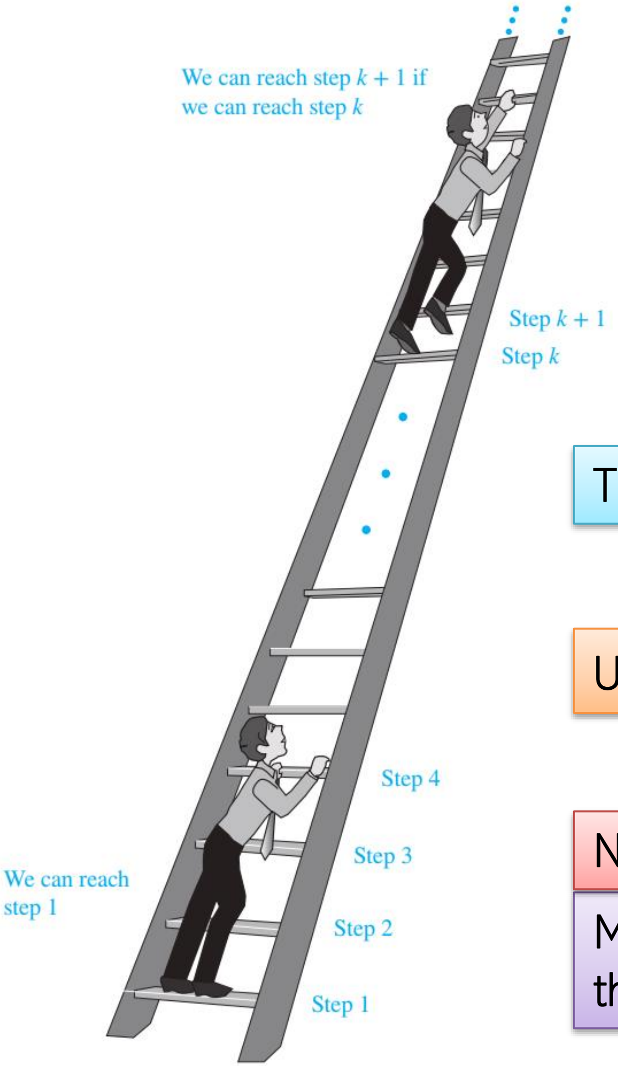
Infinite ladder

1. We can reach the first rung of the ladder.
2. If we can reach a particular rung of the ladder, then we can reach the next rung.

Therefore, we are able to reach every rung of this infinite ladder

Using proof technique called **mathematical induction**

Mathematical Induction (1/10)



Infinite ladder

1. We can reach the first rung of the ladder.
2. If we can reach a particular rung of the ladder, then we can reach the next rung.

Therefore, we are able to reach every rung of this infinite ladder

Using proof technique called mathematical induction

Note:

Mathematical induction is <u>not</u> a tool for discovering formulae or theorems.

Mathematical Induction definition:

Mathemaical induction can be used to prove statments that assert that $P(n)$ is true for all positive integers n, where $P(n)$ is a propositional function.

Principle of Mathematical Induction (1/4)

To prove that $P(n)$ is true for all positive integers n, where $P(n)$ is a propositional function,

we complete **two** steps:

Basis Step
We verify that $P(1)$ is true.

Inductive Step
We show that the conditional statment
$P(k) \rightarrow P(k+1)$ is true for all positive integers k.

Principle of Mathematical Induction (2/4)

To complete the inductive step of a proof using the principle of mathematical induction, we assume that $P(k)$ is true for an arbitrary positive integer k and show that under this assumption, $P(k+1)$ must also be true. The assumption that $P(k)$ is true is called the ***inductive hypothesis*** (**IH**).

Principle of Mathematical Induction (2/4)

To complete the inductive step of a proof using the principle of mathematical induction, we assume that $P(k)$ is true for an arbitrary positive integer k and show that under this assumption, $P(k+1)$ must also be true. The assumption that $P(k)$ is true is called the ***inductive hypothesis*** **(IH)**.

$$\forall k(P(k) \rightarrow P(k+1))$$

Principle of Mathematical Induction (3/4)

To complete the inductive step of a proof using the principle of mathematical induction, we assume that $P(k)$ is true for an arbitrary positive integer k and show that under this assumption, $P(k+1)$ must also be true. The assumption that $P(k)$ is true is called the ***inductive hypothesis*** (**IH**).

$$\forall k(P(k) \rightarrow P(k+1))$$

***Remark*:** In a proof by mathematical induction, it is **not** assumed that $P(k)$ is true for all positive integers! It is only shown that if it is assumed that $P(k)$ is true, then $P(k+1)$ is also true.

Principle of Mathematical Induction (4/4)

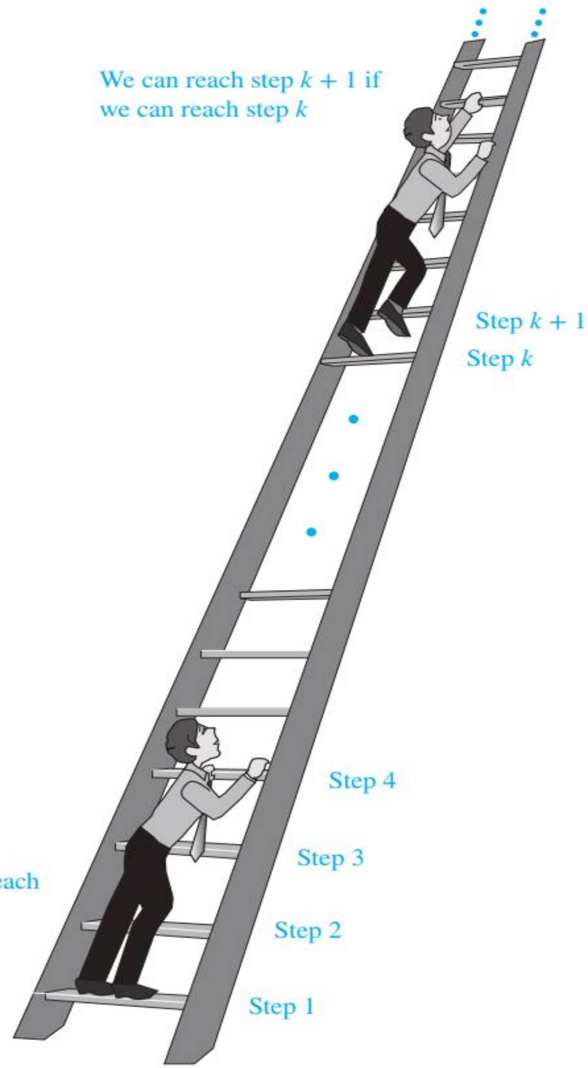
Expressed as a rule of inference,
this proof technique can be stated as:

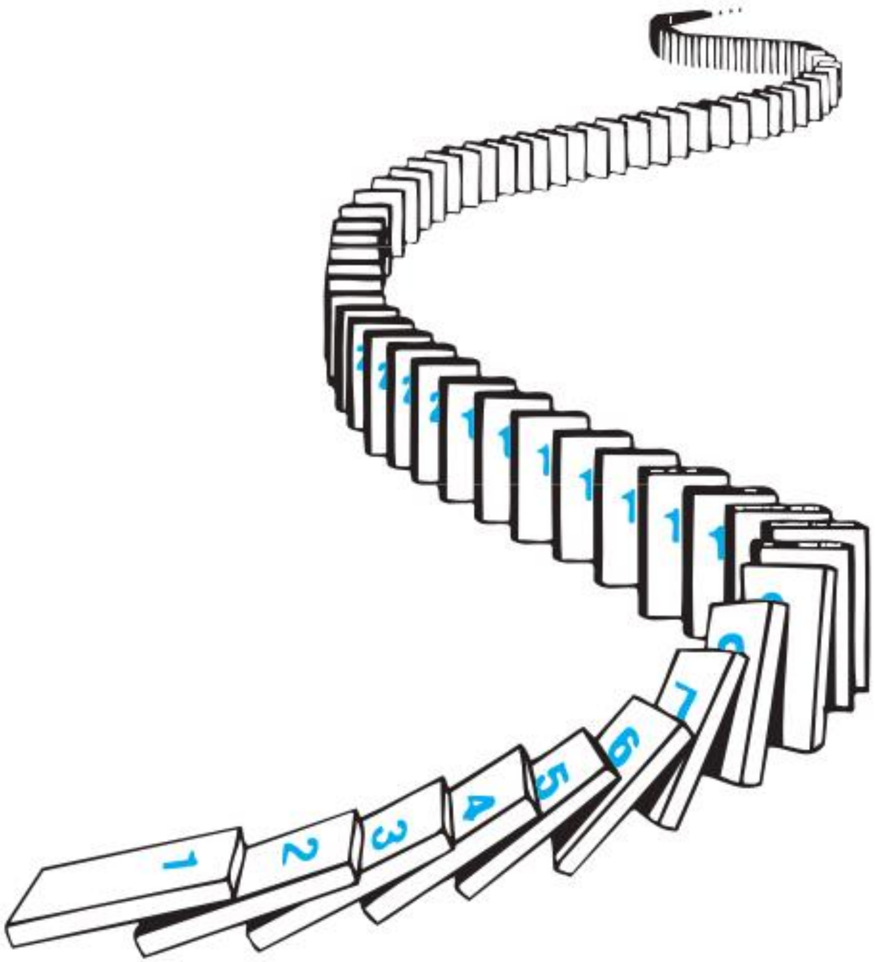
$$[P(1) \land \forall k(P(k) \rightarrow P(k+1))] \rightarrow \forall n P(n)$$

when the domain is the set of positive integers.

***Remark*:** In a proof by mathematical induction, for basis step, we **not always** start at the integer 1. In such a case, the basis step begins at a starting point *b* where *b* is an integer.

Mathematical Induction (4/10)





Notes for Proofs by Mathematical Induction (1/3)

- Express the statement that is to be proved in the form "for all $n \geq b$, $P(n)$" for a fixed integer b.
  - ✓ for all positive integers n, let $b = 1$, and
  - ✓ for all nonnegative integers n, let $b = 0$, and so on …

- Write out the words "Basis Step." Then show that $P(b)$ is true.

- Write out the words "Inductive Step" and state, and clearly identify, the inductive hypothesis, in the form "Assume that $P(k)$ is true for an arbitrary fixed integer $k \geq b$."

Mathematical Induction (5/10)

Notes for Proofs by Mathematical Induction (2/3)

- State what needs to be proved under the assumption that the inductive hypothesis (IH) is true.
  - ✓ That is, write out what $P(k+1)$ says.

- Show that $P(k+1)$ is true under the assumption that $P(k)$ is true.
  - ✓ The most difficult part of a mathematical induction proof.
  - ✓ This completes the inductive step.

Notes for Proofs by Mathematical Induction (3/3)

- After completing the basis step and the inductive step, state the conclusion, namely, “By mathematical induction, $P(n)$ is true for all integers n with $n \geq b$”.

Mathematical Induction (6/10)

Example 1:

Use mathematical induction to prove that

$$\sum_{i=1}^{n} i = 1 + 2 \ + 3 \cdots \ +n \ = \frac{n(n+1)}{2}$$

For all positive integers n . (i.e., $n \geq 1$)

Example 1 – Answer (1/4):

Let $P(n)$ be the proposition that

$$1 + 2 \ + 3 \ \cdots \ + n \ = \frac{n(n+1)}{2}$$

1) Basis Step:

If $\boldsymbol{n} \ = \ \mathbf{1}$. $P(1)$ is **true**, because $1 = \frac{(1)(2)}{2}$

This completes the basis step.

2) Inductive Step:

We first **Assume** that (Inductive Hypothesis (IH)) $P(k)$ is true for the positive integer k, $\quad$ i.e.: $P(k)$

$$"1 + 2 \ + 3 \ \cdots \ + k = \frac{k(k+1)}{2} \ ".$$

Mathematical Induction (6/10)

$P(k)$ $"1 + 2 \ + 3 \ \cdots \ + k = \frac{k(k+1)}{2} \ ".$

Example 1 – Answer (2/4):

We **need to show** that if $P(k)$ is true, then $P(k+1)$ is true.

i.e., we need to show that $P(k+1)$ is also true.

$$1 + 2 \ + 3 \ \cdots \ + k + (k+1) = \frac{(k+1)[(k+1)+1]}{2} = \frac{(k+1)(k+2)}{2}$$

Example 1 – Answer (3/4):

$P(k)$
$$"1+2\ \ +3\ \cdots +k=\frac{k(k+1)}{2}".$$

We **add** $(\boldsymbol{k}+\mathbf{1})$ to both sides of the equation in $P(k)$, we obtain

$$1+2\ \ +3\ \cdots\ +k+(k+1)\overset{\text{IH}}{=}\frac{k(k+1)}{2}+(k+1)$$

$$=\frac{k(k+1)+2(k+1)}{2}$$

$$=\frac{(k+1)(k+2)}{2}$$

Mathematical Induction (6/10)

Example 1 – Answer (3/4):

$P(k)$

$$\text{"}1 + 2 \;\; + 3 \;\cdots\; + k = \frac{k(k+1)}{2}\text{"}.$$

We **add** $(\boldsymbol{k} + \mathbf{1})$ to both sides of the equation in $P(k)$, we obtain

$$1 + 2 \;\; + 3 \;\cdots\; + k + \boxed{(k+1)} \overset{\text{IH}}{=} \frac{k(k+1)}{2} + \boxed{(k+1)}$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

- **This equation show that $P(k+1)$ is true under the assumption that $P(k)$ is true.**
- **This completes the inductive step.**

Example 1 – Answer (4/4):

So, by mathematical induction we know that $P(n)$ is true for all positive integers n.

That is, we proven that

$$1 + 2 \ + 3 \ \cdots \ + n \ = \frac{n(n+1)}{2}$$

for all positive integers n.

Mathematical Induction (7/10)

Example 2:

Use mathematical induction to prove that

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 \ + 3^2 \cdots + n^2 \ = \frac{n(n+1)(2n+1)}{6}$$

For all positive integers n . (i.e., $n \geq 1$)

Example 2 – Answer (1/4):

Let $P(n)$ be the proposition that

$$1^2 + 2^2 \ + 3^2 \ \cdots \ + n^2 \ = \frac{n(n+1)(2n+1)}{6}$$

1) Basis Step:

If $\boldsymbol{n} \ = \ \mathbf{1}$. $P(1)$ is **true**, because $1^2 = 1 = \frac{(1)(2)(3)}{6}$

This completes the basis step.

2) Inductive Step:

We first **Assume** that (Inductive Hypothesis (IH)) $P(k)$ is true for the positive integer k, $\quad$ i.e.: $P(k)$

$$"1^2 + 2^2 \ + 3^2 \ \cdots \ + k^2 = \frac{k(k+1)(2k+1)}{6} \ ".$$

Example 2 – Answer (2/4):

$P(k)$
$"1^2 + 2^2 \ + 3^2 \ \cdots \ + k^2 = \frac{k(k+1)(2k+1)}{6}"$.

We **need to show** that if $P(k)$ is true, then $P(k + 1)$ is true.

i.e.: we need to show that $P(k + 1)$ is also true.

$$1^2 + 2^2 \ + 3^2 \ \cdots \ + k^2 + (k+1)^2 = \frac{(k+1)[(k+1)+1][2(k+1)+1]}{6}$$

$$1^2 + 2^2 \ + 3^2 \ \cdots \ + k^2 + (k+1)^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

Mathematical Induction (7/10)

Example 2 – Answer (3/4):

$P(k)$
$$"1^2 + 2^2 \ + 3^2 \ \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}".$$

We **add** $(\boldsymbol{k}+\mathbf{1})^{\mathbf{2}}$ to both sides of the equation in $P(k)$, we obtain

$$1^2 + 2^2 \ + 3^2 \ \cdots + k^2 + (k+1)^2 \overset{\text{IH}}{=} \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

$$= \frac{(k+1)(k(2k+1) + 6(k+1))}{6}$$

Mathematical Induction (7/10)

Example 2 – Answer (3/4):

$P(k)$
"$1^2 + 2^2 + 3^2 \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$".

We **add** $(\boldsymbol{k+1})^{\mathbf{2}}$ to both sides of the equation in $P(k)$, we obtain

$$1^2 + 2^2 + 3^2 \cdots + k^2 + (k+1)^2 \overset{\text{IH}}{=} \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= \frac{(k+1)(k(2k+1) + 6(k+1))}{6}$$

$$= \frac{(k+1)(2k^2 + 7k + 6)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

- **This equation show that $P(k+1)$ is true under the assumption that $P(k)$ is true.**
- **This completes the inductive step.**

Example 2 – Answer (4/4):

So, by mathematical induction we know that $P(n)$ is true for all positive integers n.

That is, we proven that

$$1^2 + 2^2 \ + 3^2 \ \cdots \ + n^2 \ = \frac{n(n+1)(2n+1)}{6}$$

for all positive integers n.

Mathematical Induction (8/10)

Example 3:

Use mathematical induction to prove that

$$n < 2^n$$

For all positive integers n . (i.e., $n \geq 1$)

Example 3 – Answer (1/4):

Let $P(n)$ be the proposition that

$$n < 2^n$$

1) Basis Step:

If $\boldsymbol{n = 1}$. $P(1)$ is **true**, because $1 < 2^1$

This completes the basis step.

2) Inductive Step:

We first **Assume** that (Inductive Hypothesis (IH)) $P(k)$ is true for the positive integer k, $\quad$ i.e.: $P(k)$

$$k < 2^k$$

Mathematical Induction (8/10)

$P(k)$ | $k < 2^k$

Example 3 – Answer (2/4):

We **need to show** that if $P(k)$ is true, then $P(k+1)$ is true.

i.e., we need to show that $P(k+1)$ is also true.

$$(k+1) < 2^{k+1}$$

$P(k)$ $k < 2^k$

Example 3 – Answer (3/4):

We **add** (**1**) to both sides of the equation in $P(k)$, we obtain

$$(k + 1) \overset{\text{IH}}{<} 2^k + 1$$

Example 3 – Answer (3/4):

$P(k)$ $k < 2^k$

We **add** (**1**) to both sides of the equation in $P(k)$, we obtain

$$(k+1) \overset{\text{IH}}{<} 2^k + \boxed{1}$$

Because the integer $k \geq 1$. Therefore, $2^k > 1$

$$(k+1) < 2^k + \boxed{2^k}$$

Mathematical Induction (8/10)

Example 3 – Answer (3/4):

$P(k)$ $k < 2^k$

We **add** (**1**) to both sides of the equation in $P(k)$, we obtain

$$(k+1) \overset{\text{IH}}{<} 2^k + 1$$

$$(k+1) < 2^k + 2^k$$

$$(k+1) < 2 \cdot 2^k$$

$$(k+1) < 2^{k+1}$$

$P(k)$ $k < 2^k$

Example 3 – Answer (3/4):

We **add** ($\mathbf{1}$) to both sides of the equation in $P(k)$, we obtain

$$(k+1) \overset{\text{IH}}{<} 2^k + 1$$

$$(k+1) < 2^k + 2^k$$

$$(k+1) < 2 \cdot 2^k$$

$$(k+1) < 2^{k+1}$$

- **This equation show that $P(k+1)$ is true under the assumption that $P(k)$ is true.**
- **This completes the inductive step.**

Example 3 – Answer (4/4):

So, by mathematical induction we know that $P(n)$ is true for all positive integers n.

That is, we proven that

$$n < 2^n$$

for all positive integers n.

Mathematical Induction (9/10)

Example 4:

Use mathematical induction to prove that

$$2^n < n!$$

For every integer integers n with $n \geq 4$.

Example 4 – Answer (1/5):

Let $P(n)$ be the proposition that

$$2^n < n! \qquad n \geq 4$$

1) Basis Step:

If $\boldsymbol{n} = \mathbf{4}$. $P(4)$ is **true**, because $(2^4 = 16) < (4! = 24)$

This completes the basis step.

2) Inductive Step:

We first **Assume** that (Inductive Hypothesis (IH)) $P(k)$ is true for the positive integer $k \geq 4$, $\quad$ i.e.: $P(k)$

$$2^k < k!$$

$P(k)$ $2^k < k!$

Example 4 – Answer (2/5):

We **need to show** that if $P(k)$ is true, then $P(k+1)$ is true.

$\text{i.e., we need to show that } P(k+1) \text{ is also true.}$

$k \geq 4$

$2^{k+1} < (k+1)!$

$$2^{k+1} < (k+1)!$$

Example 4 – Answer (3/5):

$P(k)$ $2^k < k!$

$k \geq 4$

We are **multiple** both sides of the equation in $P(k)$ by $(\mathbf{2})$, we obtain

$$2^k \overset{\text{IH}}{<} k!$$

$$2 \cdot 2^k < 2 \cdot k!$$

By definition of exponent

$2^{k+1} = 2 \cdot 2^k$

Mathematical Induction (9/10)

Example 4 – Answer (3/5):

$P(k)$ $2^k < k!$

$k \geq 4$

We are **multiple** both sides of the equation in $P(k)$ by (**2**), we obtain

$$2^k \overset{\text{IH}}{<} k!$$

$$2 \cdot 2^k < 2 \cdot k!$$

By definition of exponent
$2^{k+1} = 2 \cdot 2^k$

$$2^{k+1} < 2 \cdot k!$$

Mathematical Induction (9/10)

Example 4 – Answer (4/5):

$P(k)$ $2^k < k!$

$k \geq 4$

We are **multiple** both sides of the equation in $P(k)$ by $(\mathbf{2})$, we obtain

$$2^{k+1} < \boxed{2} \cdot k!$$

Because the integer $k \geq 4$. Therefore, $2 < k + 1$

$$2^{k+1} < \boxed{(k+1)} \cdot k!$$

Mathematical Induction (9/10)

Example 4 – Answer (4/5):

$P(k)$ $2^k < k!$

$k \geq 4$

We are **multiple** both sides of the equation in $P(k)$ by $(\mathbf{2})$, we obtain

$$2^{k+1} < 2 \cdot k!$$

$$2^{k+1} < (k+1) \cdot k!$$

By definition of factorial function.

$$2^{k+1} < (k+1)!$$

$P(k)$ $2^k < k!$

$k \geq 4$

Example 4 – Answer (4/5):

We are **multiple** both sides of the equation in $P(k)$ by $(\mathbf{2})$, we obtain

$$2^{k+1} < 2 \cdot k!$$

$$2^{k+1} < (k+1) \cdot k!$$

$$2^{k+1} < (k+1)!$$

- **This equation show that $P(k+1)$ is true under the assumption that $P(k)$ is true.**
- **This completes the inductive step.**

Example 4 – Answer (5/5):

So, by mathematical induction we know that $P(n)$ is true for all positive integers $n \geq 4$.

That is, we proven that

$$2^n < n!$$

for all positive integers $n \geq 4$.

Example 5:

Use mathematical induction to prove that

$$n^3 - n \text{ is divisible by } 3$$

For every positive integer integers n. (i.e., $n \geq 1$)

Example 5 – Answer (1/4):

Let $P(n)$ be the proposition that

$$\text{"}\ n^3 - n \text{ is divisible by } 3\ \text{"} \qquad \boxed{n \geq 1}$$

1) Basis Step:

If $\boldsymbol{n} = \mathbf{1}$. $P(1)$ is **true**, because $(1^3 - 1 = 0)$ is divisible by 3.
This completes the basis step.

2) Inductive Step:

We first **Assume** that (Inductive Hypothesis (IH)) $P(k)$ is true for the positive integer $k \geq 1$, i.e.: $P(k)$

$$k^3 - k \text{ is divisible by } 3$$

$P(k)$

$k^3 - k$ is divisible by 3

Example 5 – Answer (2/4):

We **need to show** that if $P(k)$ is true, then $P(k+1)$ is true.

i.e., we need to show that $P(k+1)$ is also true.

$$(k+1)^3 - (k+1) \text{ is divisible by } 3$$

$P(k)$

$k^3 - k$ is divisible by 3

Example 5 – Answer (3/4):

Note that

$$(k+1)^3 - (k+1) = (k^3 + 3k^2 + 3k + 1) - (k+1)$$

$$= k^3 + 3k^2 + 3k - k$$

$$= k^3 - k + 3k^2 + 3k$$

$$= (k^3 - k) + 3(k^2 + k)$$

$P(k)$

$k^3 - k$ is divisible by 3

Example 5 – Answer (3/4):

Note that

$$(k+1)^3 - (k+1) = (k^3 + 3k^2 + 3k + 1) - (k+1)$$

$$= k^3 + 3k^2 + 3k - k$$

$$= k^3 - k + 3k^2 + 3k$$

$$= (k^3 - k) + 3(k^2 + k)$$

Using the inductive hypothesis, we conclude that the first term $k^3 - k$ is divisible by 3

$P(k)$

$k^3 - k$ is divisible by 3

Example 5 – Answer (3/4):

Note that

$$(k+1)^3 - (k+1) = (k^3 + 3k^2 + 3k + 1) - (k+1)$$

$$= k^3 + 3k^2 + 3k - k$$

$$= k^3 - k + 3k^2 + 3k$$

$$= (k^3 - k) + \boxed{3(k^2 + k)}$$

The second term is divisible by 3 because it is 3 times an integer.

$P(k)$

$k^3 - k$ is divisible by 3

Example 5 – Answer (3/4):

Note that

$$(k+1)^3 - (k+1) = (k^3 + 3k^2 + 3k + 1) - (k+1)$$

$$= k^3 + 3k^2 + 3k - k$$

$$= k^3 - k + 3k^2 + 3k$$

$$= (k^3 - k) + 3(k^2 + k)$$

- **So, $(k+1)^3 - (k+1)$ is divisible by 3**
- **This completes the inductive step.**

Example 5 – Answer (4/4):

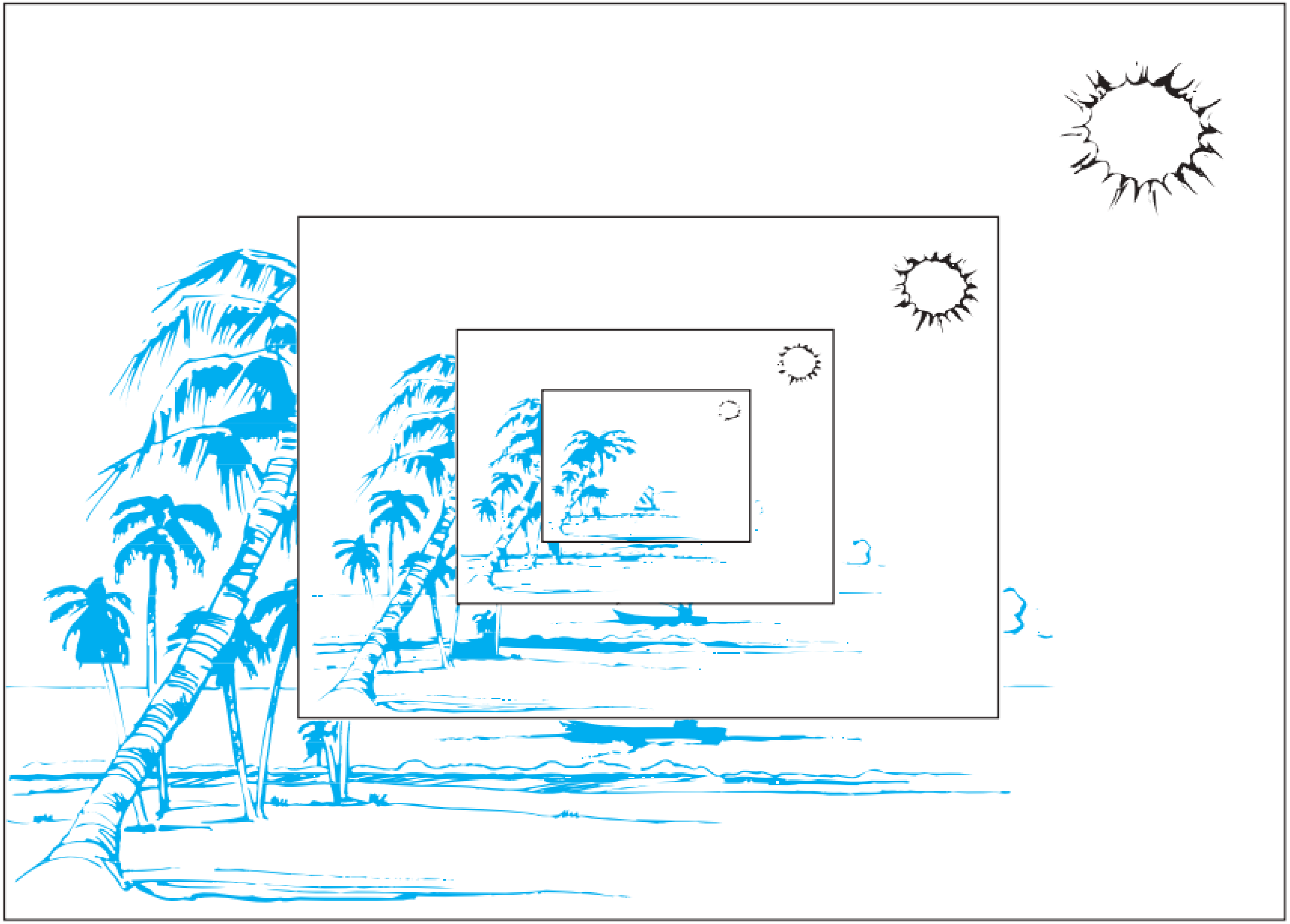
So, by mathematical induction we know that $P(n)$ is true for all positive integers $n \geq 1$.

That is, we proven that

" $n^3 - n$ is divisible by 3 "

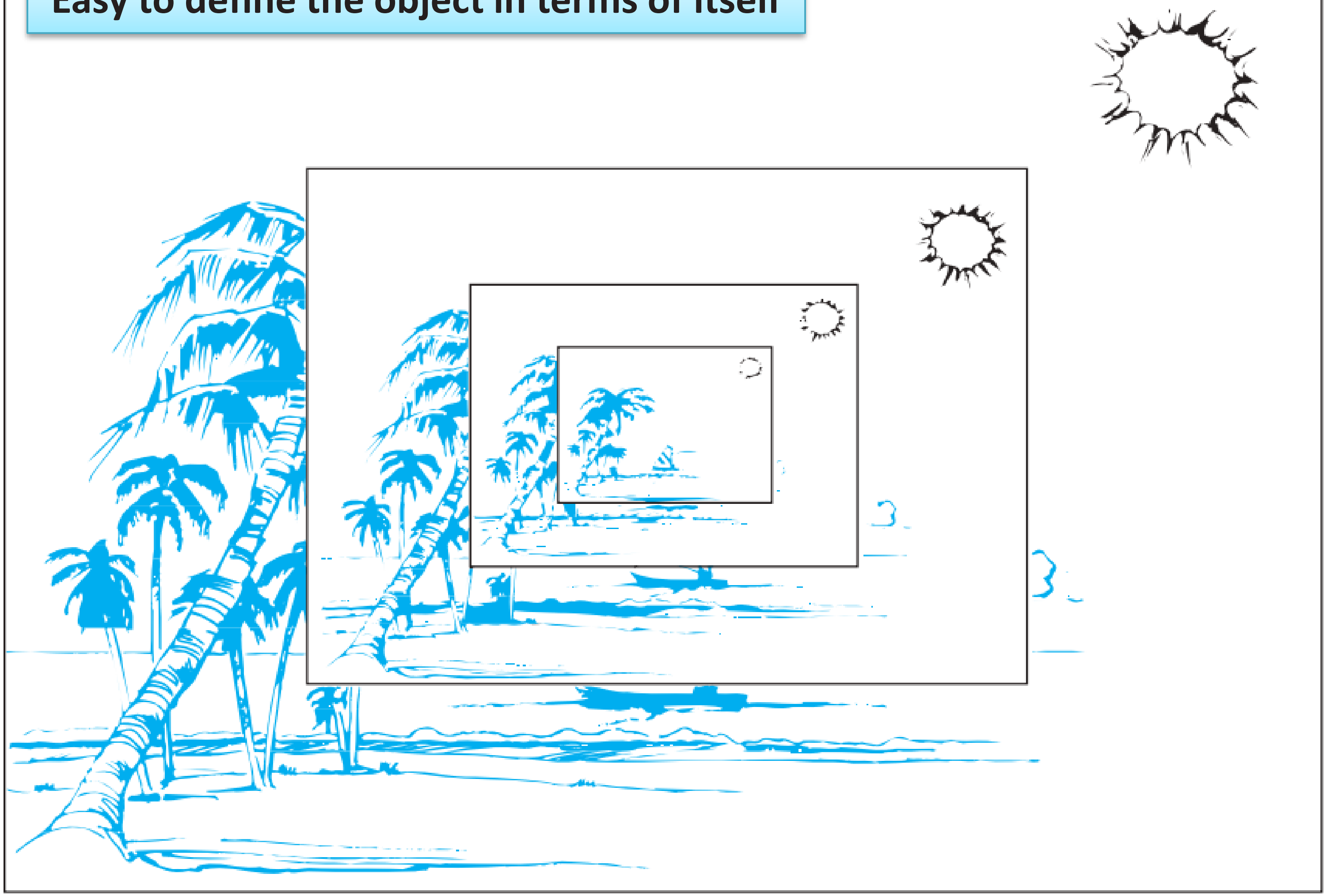
for all positive integers $n \geq 1$.

Recursive Definitions (1/13)



Recursive Definitions (1/13)

Easy to define the object in terms of itself



Recursive Definitions (2/13)

Recursion:

The process of defining an object in terms of itself.

Recursively Defined Functions:

Basis Step

Specify the value of the function at the first point.

Recursive Step

Specifying how terms in the function are found from previous terms.

Recursive Definitions (3/13)

Example 1:

We use two steps to define a function with the set of *nonnegative integers* as its domain:

1) Basis Step:

Specify the value of the function at *zero*.

$f(0) = 0$

2) Recursive Step:

Give a rule for finding its value at an integer from its values at smaller integers.

$f(n+1) = f(n) + 1,$ for integer $n \geq 0$ (i.e., nonnegative integers)

Recursive Definitions (4/13)

Example 2:

The sequence of powers of 2 is given by $a_n = 2^n$ for $n = 0, 1, 2, \ldots$

Recursive Definitions (4/13)

Example 2:

Explicit Formula

The sequence of powers of 2 is given by $a_n = 2^n$ for $n = 0, 1, 2, \ldots$

Recursive Definitions (4/13)

Example 2 – Answer:

Explicit Formula

The sequence of powers of 2 is given by $a_n = 2^n$ for $n = 0, 1, 2, \ldots$

1) Basis Step:

Specify the value of the sequence at *zero*.

$a_0 = 2^0 = 1$

2) Recursive Step:

Give a rule for finding a term of the sequence from the previous one.

$a_{n+1} = 2a_n$, for $n = 0, 1, 2, \ldots$

Example 2 – Answer:

Explicit Formula

The sequence of powers of 2 is given by $a_n = 2^n$ for $n = 0, 1, 2, \ldots$

1) Basis Step:

Specify the value of the sequence at *zero*.

$a_0 = 2^0 = 1$

2) Recursive Step:

Give a rule for finding a term of the sequence from the previous one.

$a_{n+1} = 2a_n$, for $n = 0, 1, 2, \ldots$

Recursive Formula

Recursive Definitions (5/13)

Example 3:

Suppose that f is defined recursively by

$$f(0) = 3,$$
$$f(n+1) = 2f(n) + 3.$$

Find $f(1)$, $f(2)$, $f(3)$, and $f(4)$.

Recursive Definitions (5/13)

Example 3 – Answer:

Suppose that f is defined recursively by

$$f(0) = 3,$$
$$f(n+1) = 2f(n) + 3.$$

Find $f(1)$, $f(2)$, $f(3)$, and $f(4)$.

Solution: From the recursive definition it follows that

$$f(1) = 2f(0) + 3 = 2 \cdot 3 + 3 = 9,$$
$$f(2) = 2f(1) + 3 = 2 \cdot 9 + 3 = 21,$$
$$f(3) = 2f(2) + 3 = 2 \cdot 21 + 3 = 45,$$
$$f(4) = 2f(3) + 3 = 2 \cdot 45 + 3 = 93.$$

Recursive Definitions (6/13)

Example 4:

Give a recursive definition of the factorial function $n!$

Recursive Definitions (6/13)

Example 4 – Answer:

Give a recursive definition of the factorial function $n!$

1) Basis Step:

Specify the value of the function at *zero*.

$f(0) = 1$

2) Recursive Step:

Give a rule for finding its value at an integer from its values at smaller integers.

$f(n+1) = (n+1) \cdot f(n) \quad , \qquad \text{for } n = 0, 1, 2, \ldots$

Example 5:

Recall from Chapter 2 that the Fibonacci numbers, $f_0, f_1, f_2, \ldots$, are defined by the equations $f_0 = 0$, $f_1 = 1$, and

$$f_n = f_{n-1} + f_{n-2}$$

Find:

f_2

f_3

f_4

f_5

Recursive Definitions (7/13)

Example 5 – Answer:

Recall from Chapter 2 that the Fibonacci numbers, $f_0, f_1, f_2, \dots$, are defined by the equations $f_0 = 0$, $f_1 = 1$, and

$$f_n = f_{n-1} + f_{n-2}$$

<u>Find:</u>

$$f_2 = f_1 + f_0 = 1 + 0 = 1$$

$$f_3 = f_2 + f_1 = 1 + 1 = 2$$

$$f_4 = f_3 + f_2 = 2 + 1 = 3$$

$$f_5 = f_4 + f_3 = 3 + 2 = 5$$

Recursive Definitions (8/13)

Example 6:

Give a recursive definition of

$$\sum_{k=0}^{n} a_k.$$

Recursive Definitions (8/13)

Example 6 – Answer :

Solution: The first part of the recursive definition is

$$\sum_{k=0}^{0} a_k = a_0.$$

The second part is

$$\sum_{k=0}^{n+1} a_k = \left(\sum_{k=0}^{n} a_k\right) + a_{n+1}.$$

Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz

Lectures #6: https://www.youtube.com/watch?v=E8KWOSBQSuE&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=36

https://www.youtube.com/watch?v=xKzYNC1cPZk&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=37

https://www.youtube.com/watch?v=ST5h-I68SLU&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=39

https://www.youtube.com/watch?v=Ov5v3IFFeQs&list=PLxIvc-MGOs6gZlMVYOOEtUHJmfUquCjwz&index=40

Thank You

Dr. Ahmed Hagag
ahagag@fci.bu.edu.eg